2022 A&A RESEARCH SHOWCASE

STUDENTS: Spencer Kraisler, RAIN Lab

Distributed Consensus Algorithms on Lie Groups

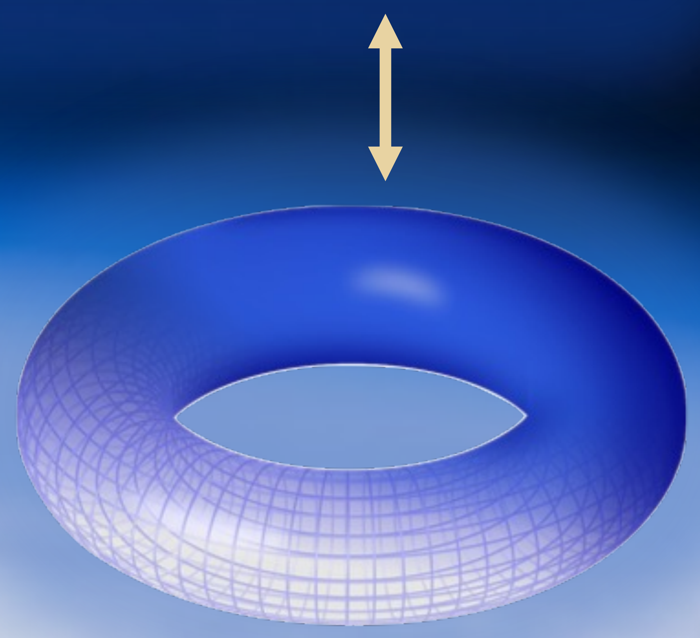
Background:

- Consensus algorithms are **inspired by nature**
- Lie Groups capture nature in fundamental ways
  - Ex. Subatomic particles

Fireflies reaching **consensus** in their blinking patterns

Motivation:

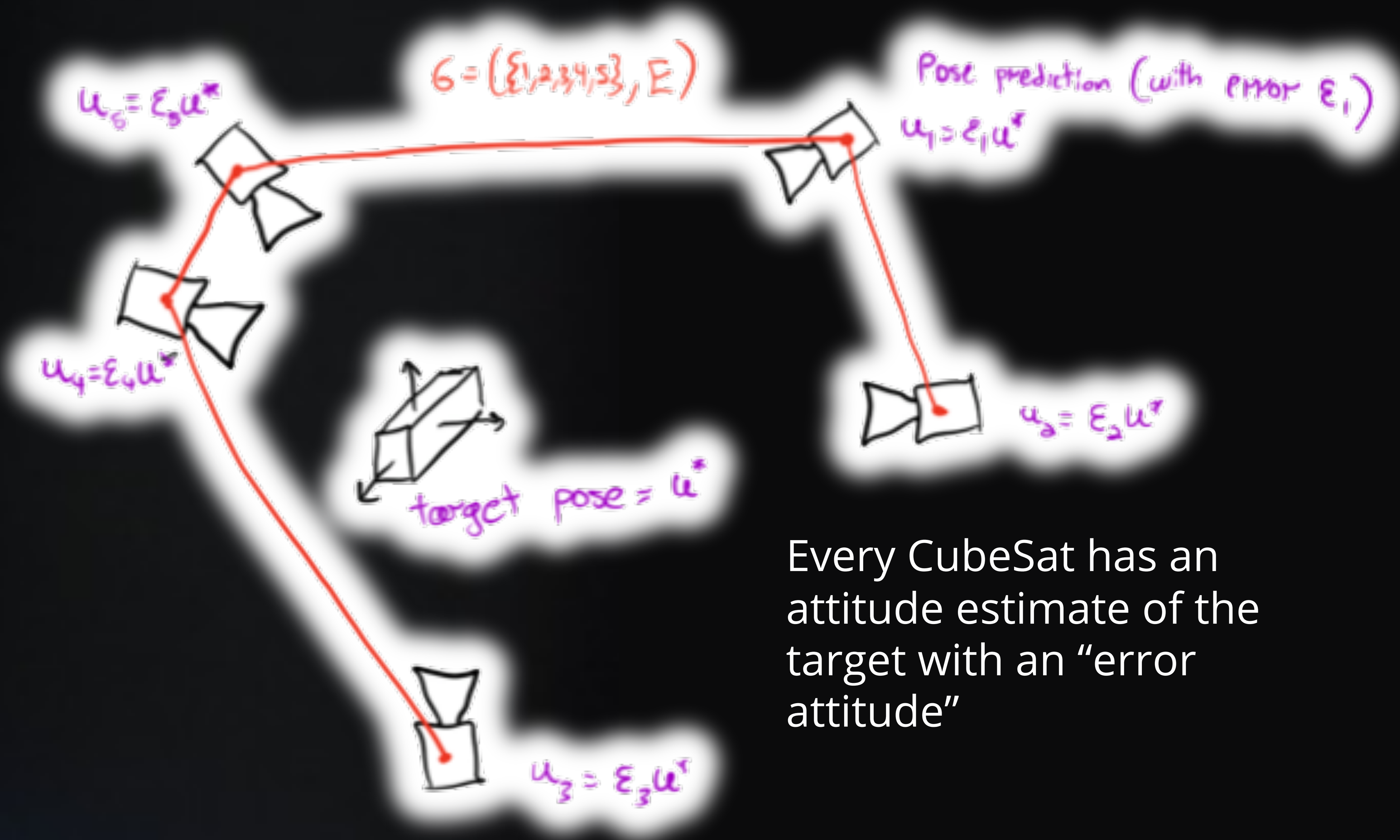
- Linearization has **limits**
- Average human paradox
- Control tools + concepts generalize naturally to Lie groups
- Very computer-friendly



The state space of a dome camera on a UAV is the Lie group $\mathbb{S}^1 \times \mathbb{S}^1$(Torus)

Applications:

- Distributed pose estimation of space debris by a network of CubeSats (**I'm doing this**)
- Coordinated motion of any system on a Lie group (ex. robot arms)



Every CubeSat has an attitude estimate of the target with an "error attitude"

Video credit:
Fireflies:
https://youtu.be/ZGvtnE1Wy6U

Drone:
https://www.youtube.com/watch?v=Ddeht8prpJw&t=21s

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Distributed Consensus Algorithms on Lie Groups

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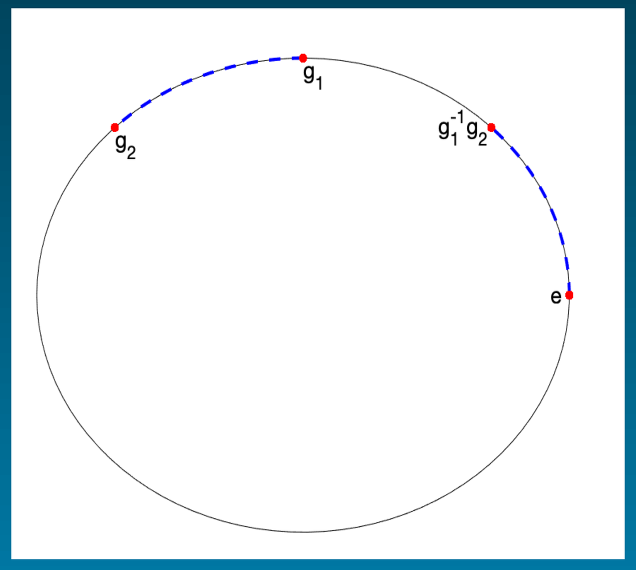
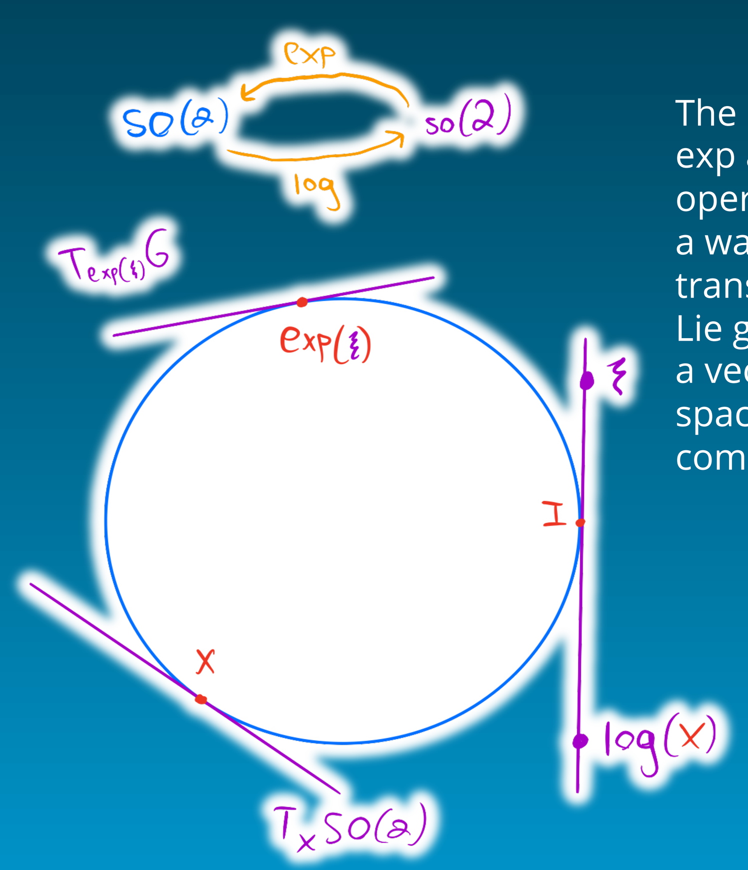
Consensus algorithms are a ubiquitous tool for multi-agent systems and distributed estimation. However, consensus algorithms are usually applied to vector spaces. For many applications, it is more natural to utilize the Lie group structure of the system than linearizing about a point. Lie groups are closed matrix groups, such as the space of 3D rotations called SO(3). I am researching ways to generalize consensus algorithms to Lie groups and apply the concept to distributed sensor networks, like a CubeSat constellation.

Review of Lie Groups

	Vector Space	Lie Groups (ex. SO(3))
Distance (squared)	$d^2(x_i, x_j) = \|\|x_j - x_i\|\|_2^2$	$d^2(g_i, g_j) = -\frac{1}{2}\mathrm{Tr}(\log(g_i^T g_j)^2)$
Relative displacement	$x_{ij} = x_j - x_i$	$g_{ij} = g_i^{-1} g_j$
Average	$\frac{1}{N}\sum_{i=1}^{N} x_i$	$\underset{g\in G}{\mathrm{argmin}} \sum_{i=1}^{N} d^2(g, g_i)$
Consensus protocol	$\dot{x}_i = \sum_{j\in N_i} x_j - x_i$	$\dot{g}_i = g_i \sum_{j\in N_i} \log(g_i^{-1} g_j)$
Point of convergence	The Euclidean average	Close to Riemannian average



The relative displacement of g_2 relative to g_1 is $g_1^{-1}g_2$



The matrix exp and log operators are a way to transform a Lie group to a vector space for computation

Consensus on Lie Groups

Lie group consensus

1. Initialize each node with $g_{i,0} \in G$
2. For each node $i = 1,\dots,N$ in parallel:
   A. Initialize the node with local measurement $g_i(0) = g_{i,0}$
   B. For $l \in \mathbb{N}$, compute the update
   $$g_i(l+1) = \exp_{g_i(l)}\left[-\epsilon(l)\sum_{j\in N_i} \log_{g_i(l)}(g_j(l))\right]$$

Euclidean consensus

1. Initialize each node with $u_i \in \mathbb{R}^n$
2. For each node $i = 1,\dots,N$ in parallel:
   A. Initialize the node with local measurement $x_i(0) = u_i$
   B. For $l \in \mathbb{N}$, compute the update
   $$x_i(l+1) = x_i(l) + \epsilon(l)\sum_{j\in N_j} (x_j(l) - x_i(l))$$

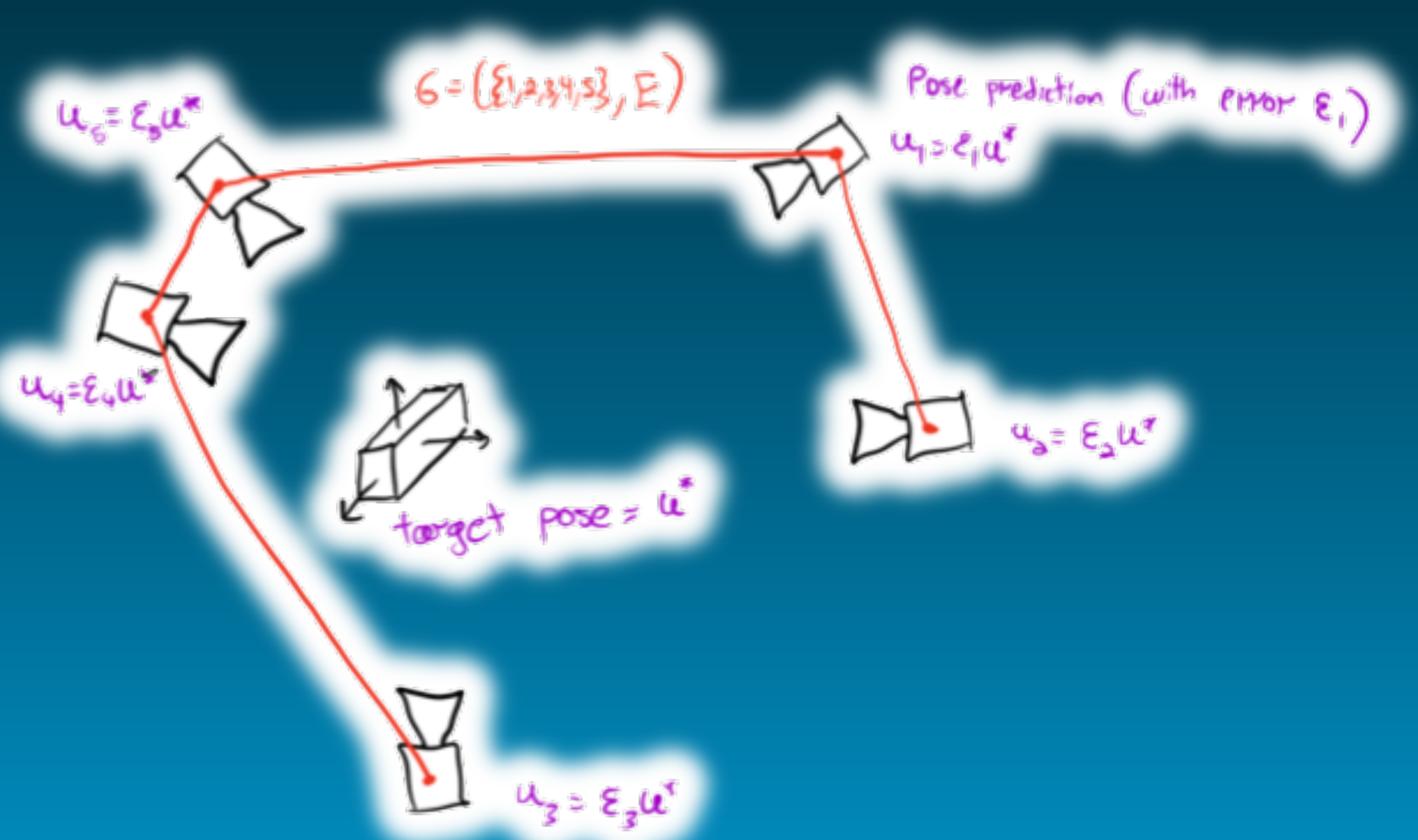
Objective

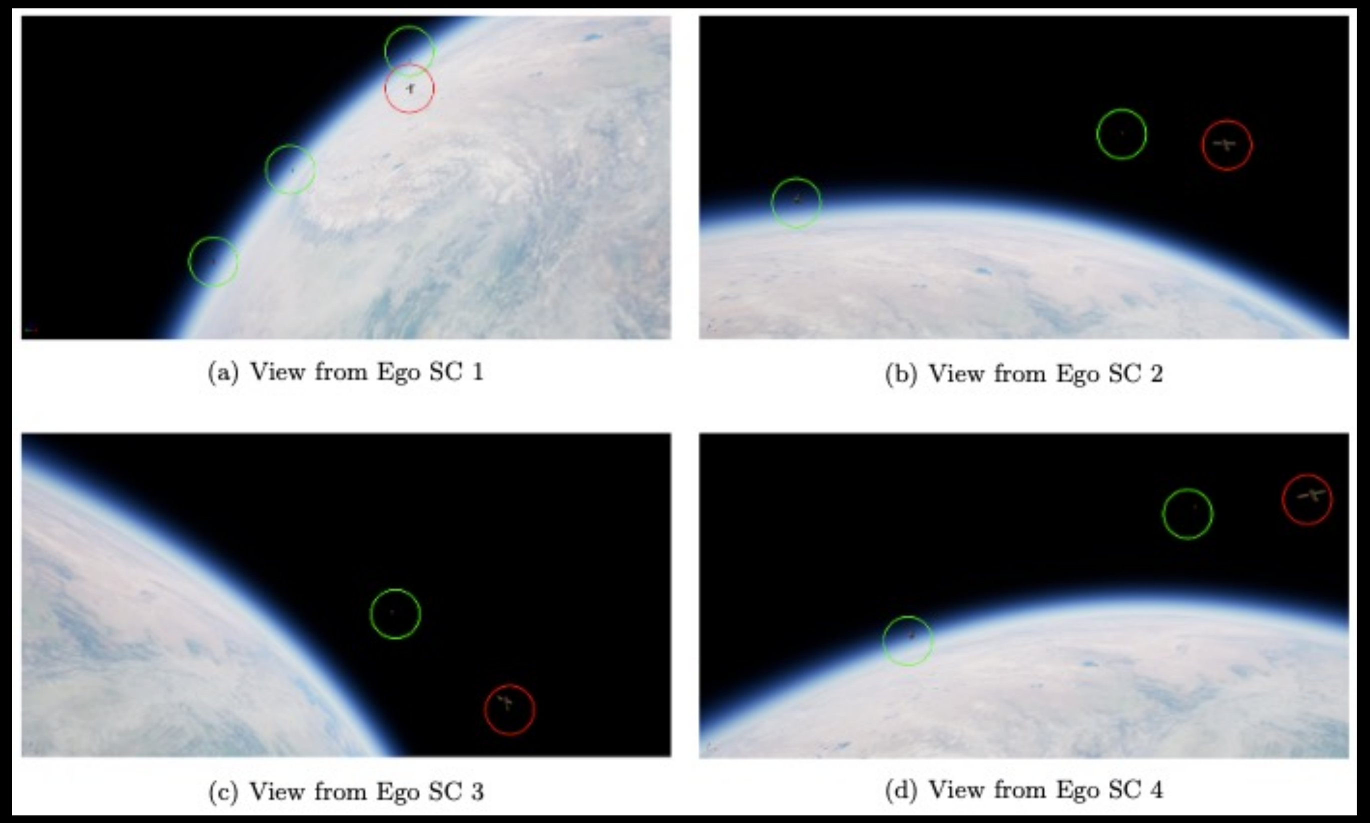
Given:

- We have a target object (red) and N controllable satellites (ego, green) with cameras and within a few kilometers of the target
- Ego satellites are under a communication network (**not** all-to-all), where each satellite can only communicate data to its "neighbors"
- Ego satellites are equipped with CNNs that estimate attitude

Goal:

- Use consensus algorithms on SO(3) for the sensor network to compute the "average" attitude of the target in a distributed fashion

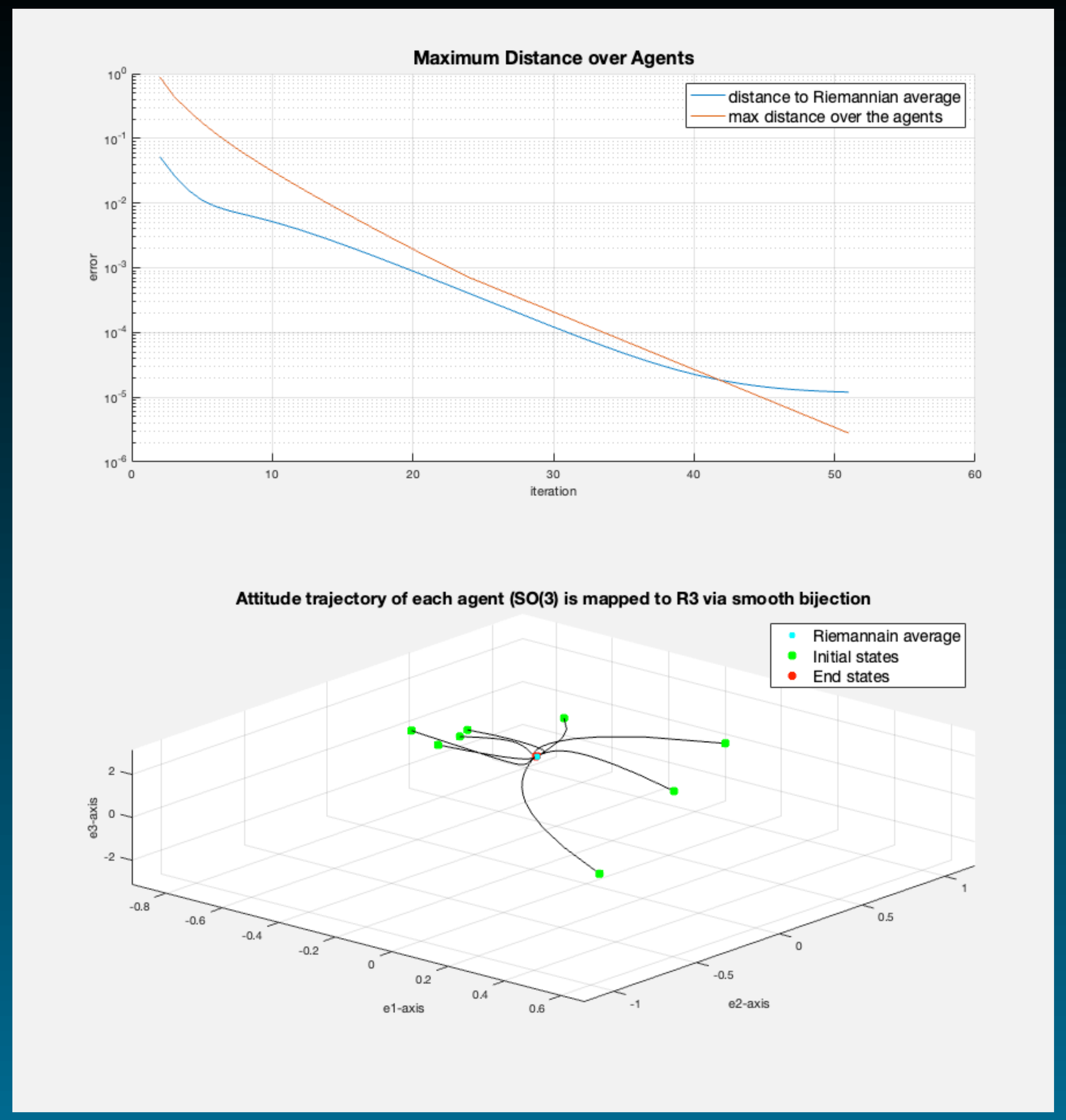




Images from our homemade unreal engine simulation. For each ego satellite, the relative attitude of the target R_{target} to the ego satellite R_i is $R_i^T R_{target}$

Results

- Big result: If the agents are initialized within a convex ball, consensus is always achieved
- Consensus point is very close to the Riemannian average!



Future Work, References, and Acknowledgments

- I am working on a general theory for consensus algorithms on Lie groups. This is one of the many possible applications
- I proved nice convergence properties of a consensus algorithm for all Lie groups. Writing a paper for 2023 IEEE Aerospace Conference

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